9

Quantum transport

Quantum transport

EPFL

Introduction to Nanoscience
S.M. Lindsay
Oxford University Press

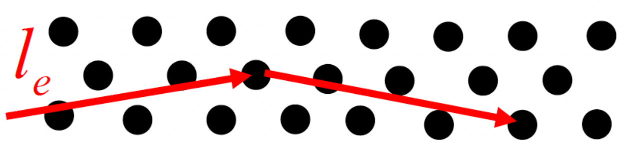
Quantum Transport: Atom to Transistor
S. Datta
Cambridge University Press

Electron Transport in Quantum Dots
L. P. Kouwenhoven et al.
Dots_Review.pdf

Quantum transport

- Transport through a 1D wire
- Coulomb blockade
- Single electron transistor

Conductivity of a macroscopic sample



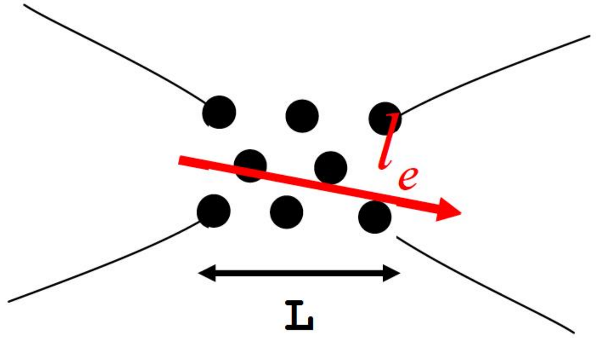
Drude / Sommerfeld model

$$\sigma = \frac{ne^2}{m^*}\frac{l_e}{v_F}$$

n : charge carrier density
m^*: effective carrier mass
l_e : carrier mean free path
v_F : Fermi velocity

Example: for copper l_e $\sim$ 30 nm

Conductivity of a nanoscopic sample

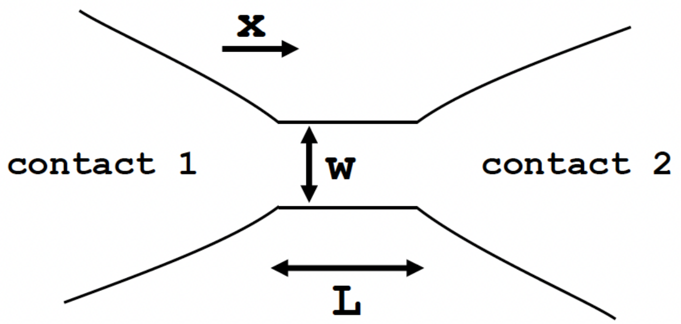


Drude model breaks down for
$l_e > L$

“ballistic conductance”

→ infinite conductance ?

1D wire



$L < l_e$ and $w < \lambda_F$

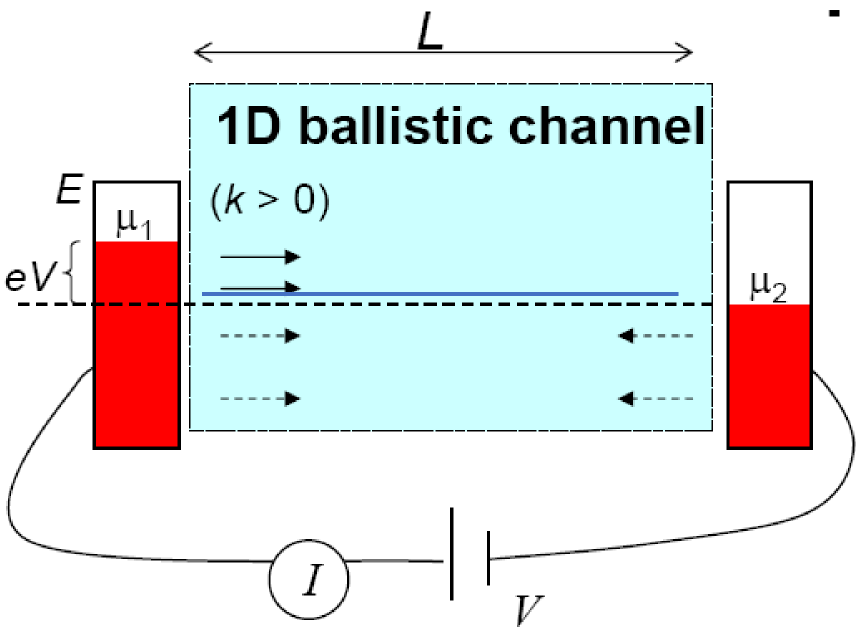
1D wire ($w \ll L$)
confinement in two directions (y and z) →
allowed energies in y and z-direction are quantized

we consider that there is one discrete state (channel) with the energy E_α in the energy range of interest eV where V is the voltage applied between the two contacts

energy of one electron $$E = E_\alpha + \frac{\hbar^2 k_x^2}{2m^*}$$

velocity along x $$v_\alpha = \frac{\hbar k_x}{m^*} = \sqrt{\frac{2}{m^*}(E - E_\alpha)}$$

Landauer formula



current ∝ charge × electron density × velocity

$$I_\alpha = e \int_{\mu_1}^{\mu_2} g_\alpha(E)\, v_\alpha(E)\, dE \qquad T = 0$$

density of states of a 1D system $\quad g_\alpha(E) \propto \sqrt{\frac{m^*}{2(E-E_0)}}$

velocity vs energy $\quad v_\alpha(E) = \sqrt{\frac{2(E-E_\alpha)}{m^*}}$

μ is the chemical potential (the Fermi level)

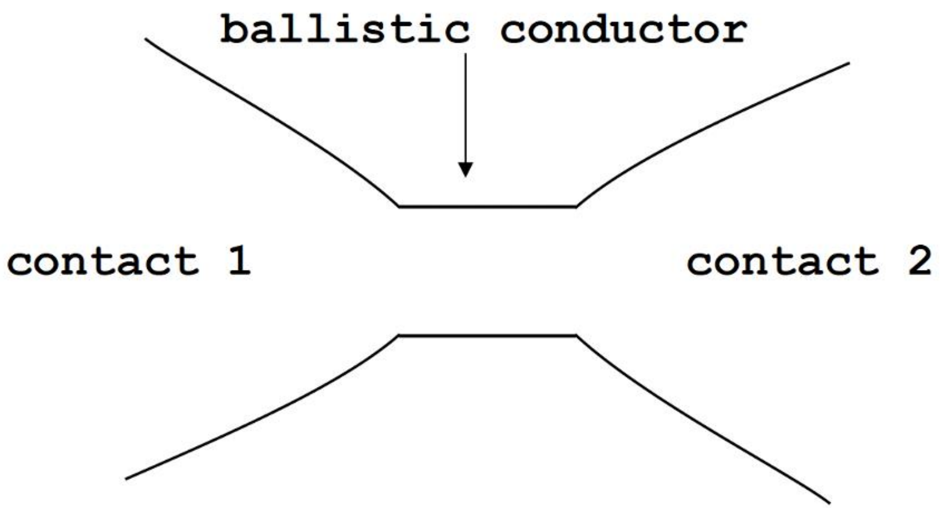
considering only $k > 0$ and a factor 2 for the spin

$$I_\alpha = e\,\frac{2}{h}(\mu_2 - \mu_1) = \frac{2e}{h}eV$$

Conductance independent of the length

$$G = \frac{I}{V} = \frac{2e^2}{h}$$

Interpretation of the conductance quantum

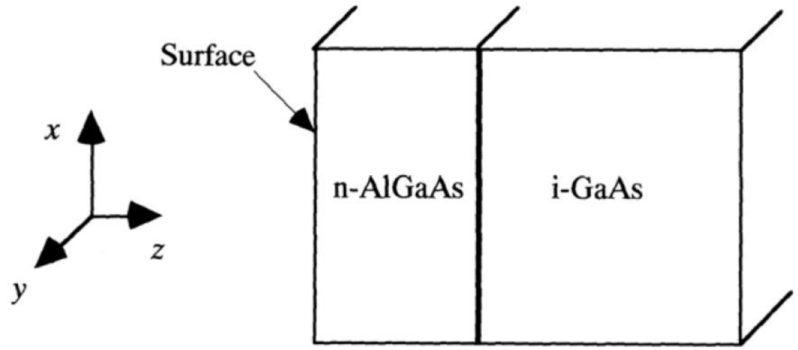


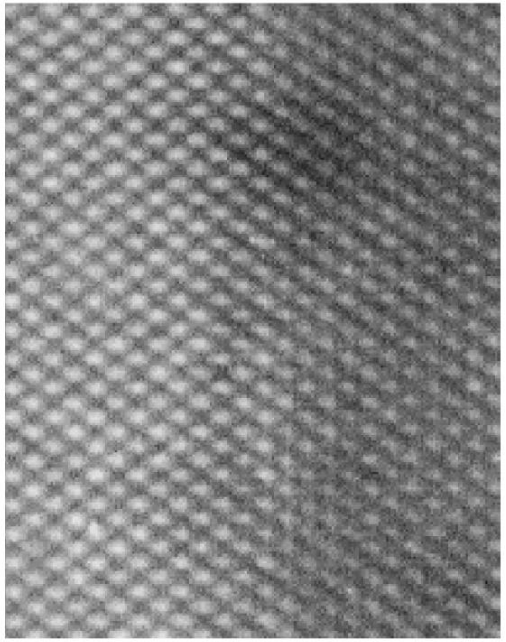
$$G_0 = \frac{2e^2}{h}$$

The conductance quantum can be rationalized as the **boundary resistance** between a perfect conductor and the contacts:
the charge carriers must scatter into the ballistic conductor from the contacts.

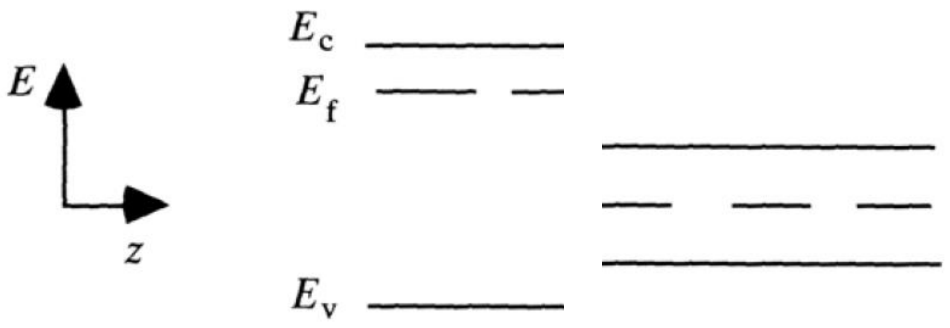
The quantum Hall effect can be used to precisely measure the conductance quantum value

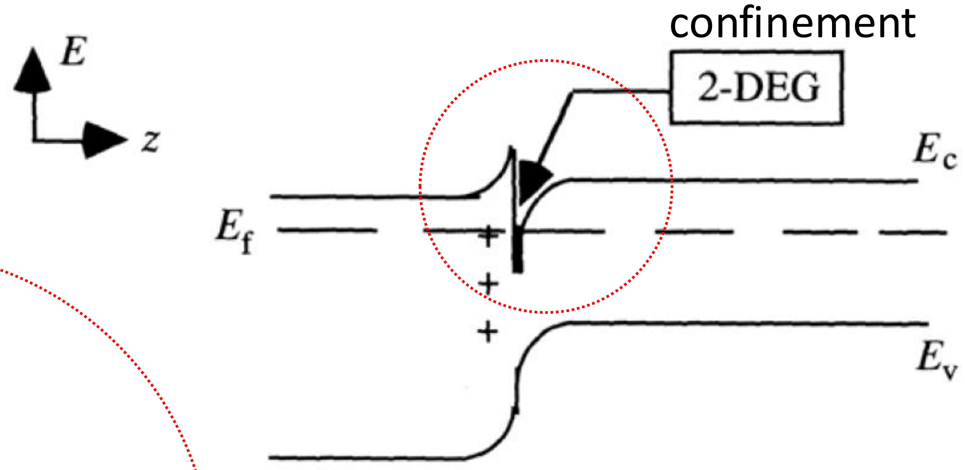
Experimental realization: 2DEG quantum point contact

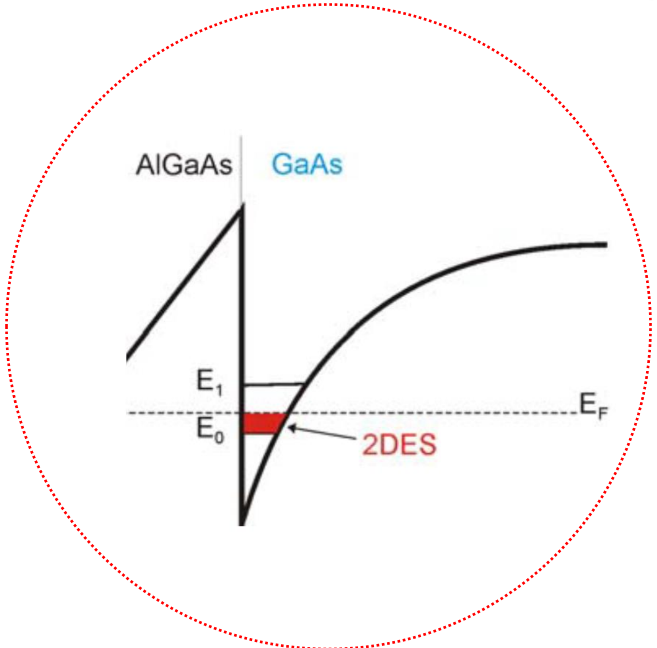




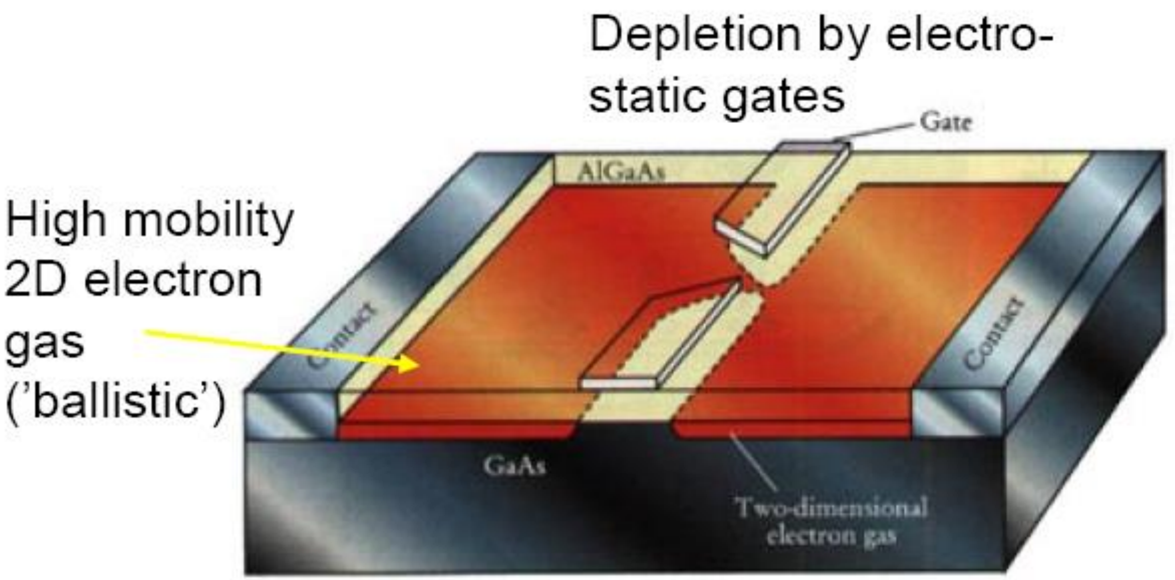
TEM image



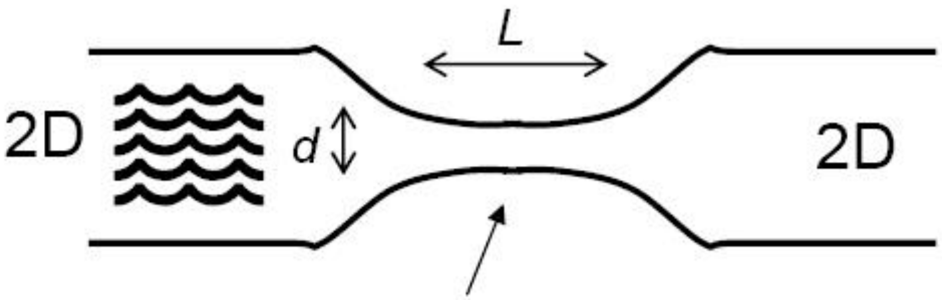




Experimental realization: 2DEG quantum point contact

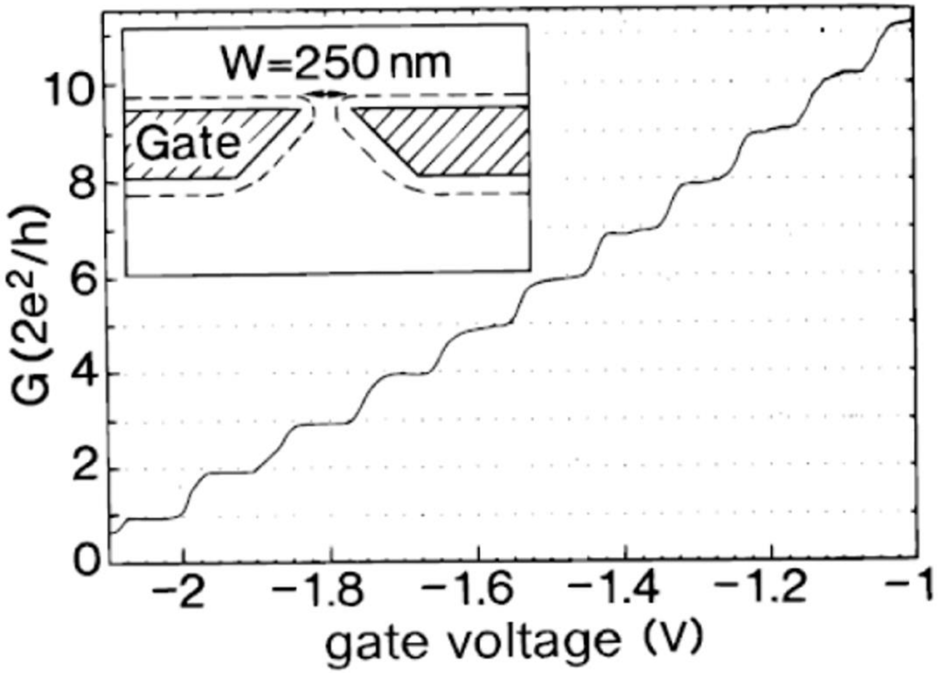


'Quantum Point Contact'



Narrow constriction; quasi-1D
(width d ~ Fermi wavelength λ_F)

d =250 nm; L =1000 nm



contribution of more channels when reducing the gate voltage (increasing the width of the point contact)

Phys. Rev. Lett. **60**, 848 (1988)

Experimental realization: wire of gold atoms

Quantized conductance through individual rows of suspended gold atoms

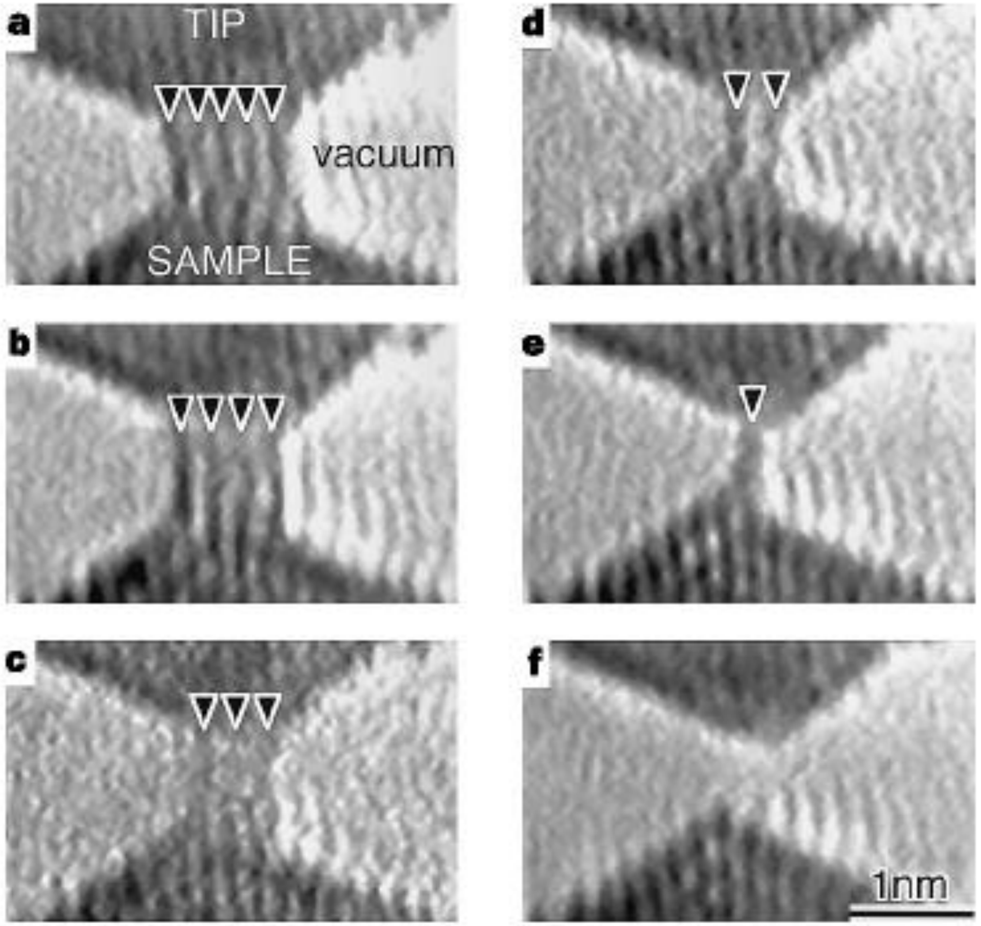


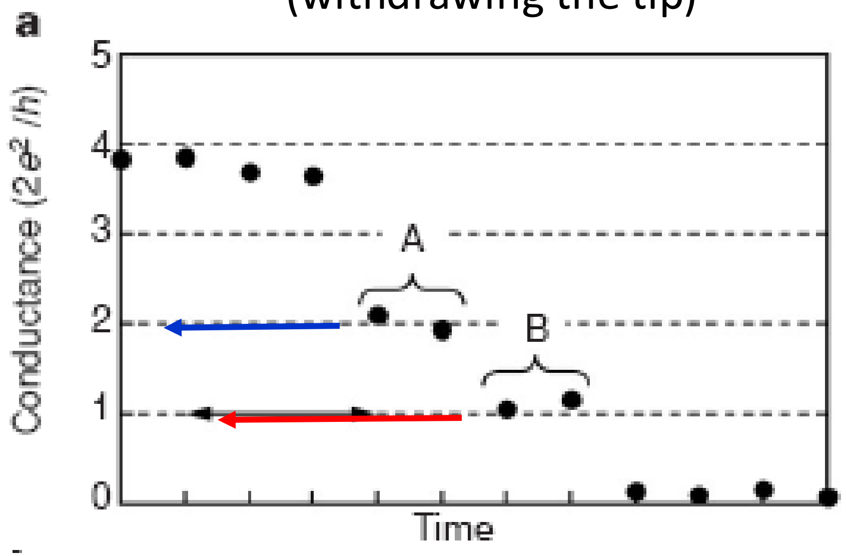
Figure 2 Electron microscope images of a contact while withdrawing the tip. A gold bridge formed between the gold tip (top) and gold substrate (bottom), thinned from **a** to **e** and ruptured at **f**, with observation times of 0, 0.47, 1.23, 1.33, 1.80 and 2.17 s, respectively. Dark lines indicated by arrowheads are rows of gold atoms. The faint fringe outside each bridge and remaining in **f** is a ghost due to interference of the imaging electrons. The conductance of the contact is 0 at **f** and $\sim 2 \times (13\,k\Omega)^{-1}$ at **e**. $V_b = -10$ mV and $R_F = 10\,k\Omega$.

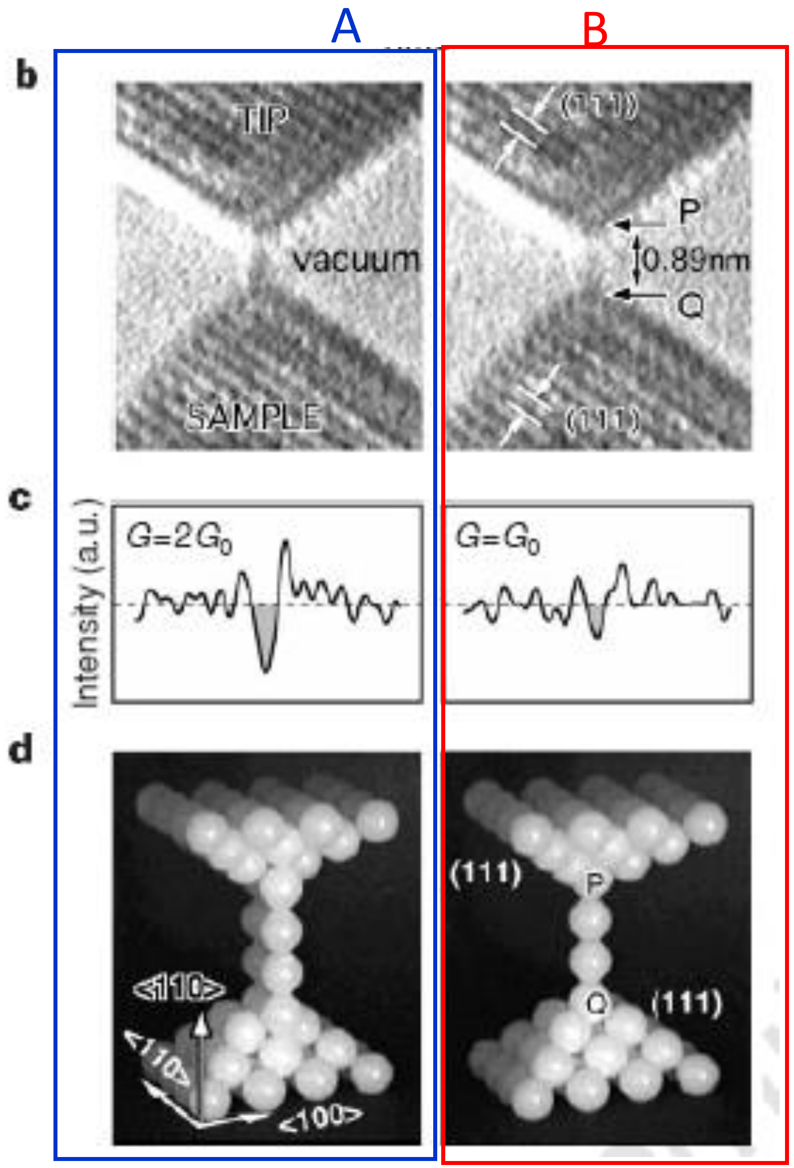
Nature **395**, 781 (1998)

The conductance is quantized: current is carried by a finite number of states (channels) in the narrow conductor

2 channels (states n=1, n=2) for the two-atom wide wire
1 channel (state n=1) for the single-atom wide wire

while pulling apart (withdrawing the tip)

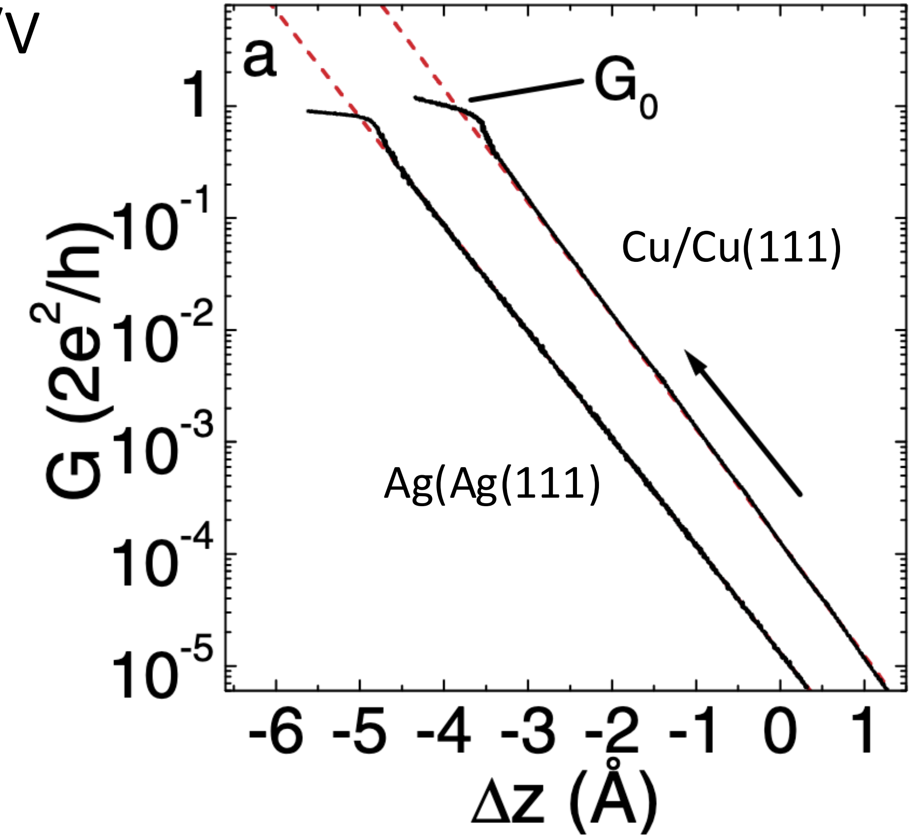


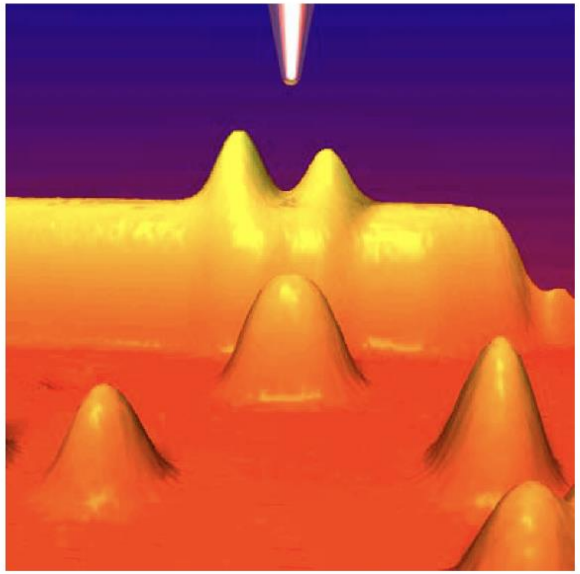


tip approaching an adatom (open feedback loop, current increases)

smooth and reproducible transition from tunneling to contact regime

G= I/V

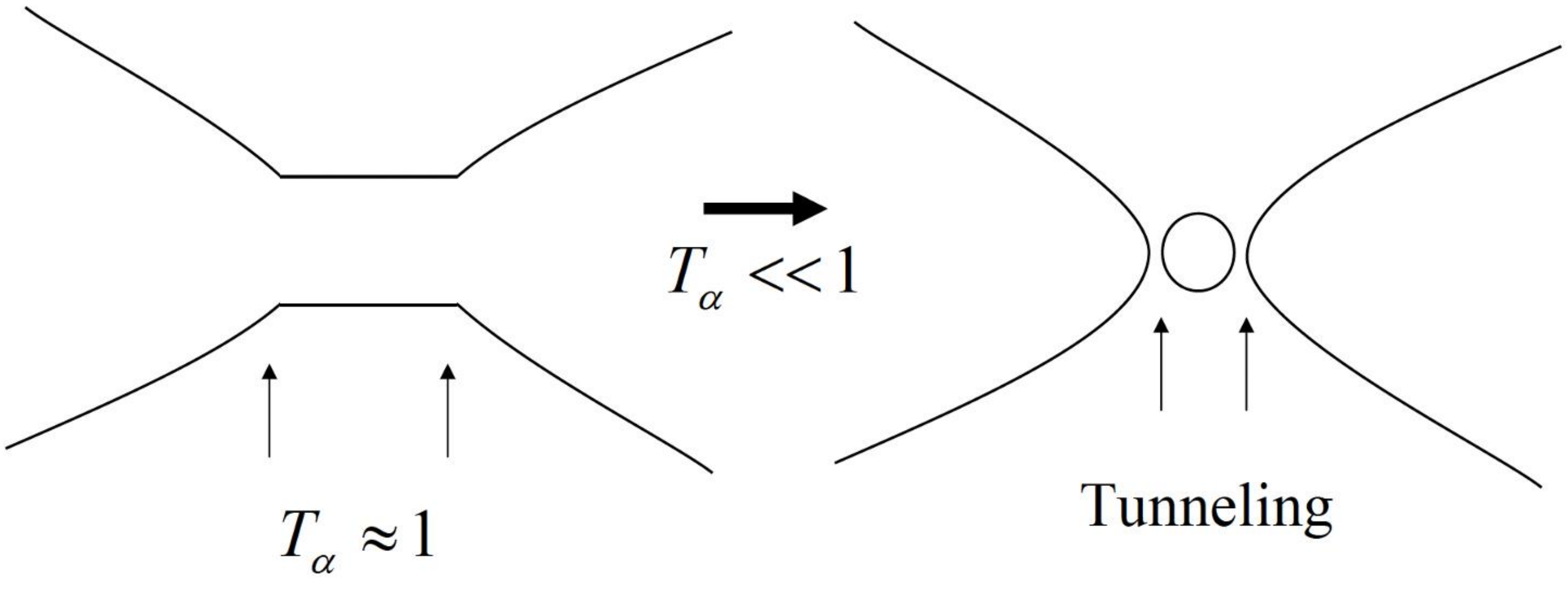


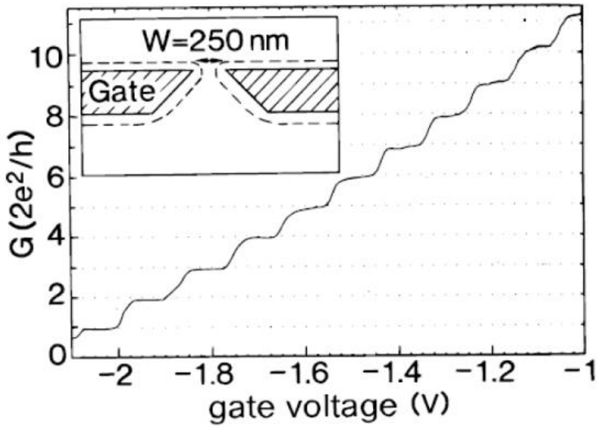


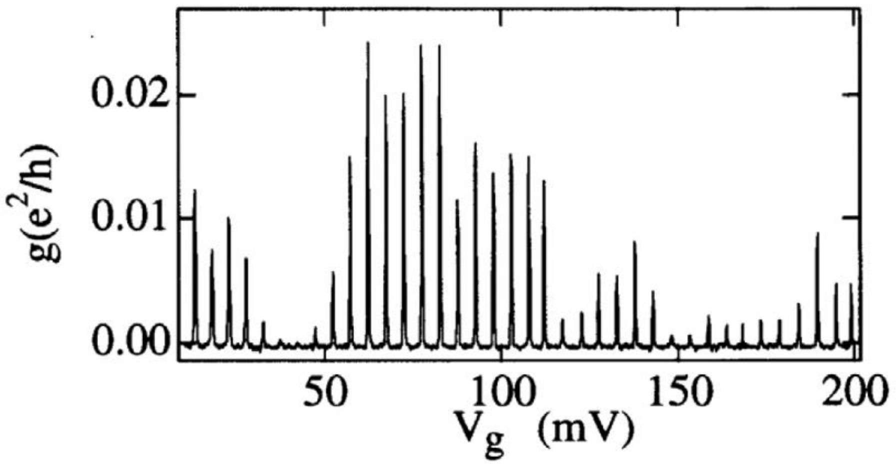
Phys. Rev. Lett. **94**, 126102 (2005)
DOI: https://doi.org/10.1103/PhysRevLett.94.126102

Reducing transmission probability

T_α: Transmission of channel α







Phys. Rev. Lett. **60**, 848 (1988)

Phys. Rev. Lett. **76** 1699 (1996)

Quantum dot: energy to add an electron

Electrons confined to small sizes

At which length scales the e-e Coulomb repulsion becomes important or dominant?

Consider a small metallic particle (dot) of diameter D.
The capacitance of this dot is $C = 4\pi\varepsilon_0\varepsilon_r\frac{D}{2}$

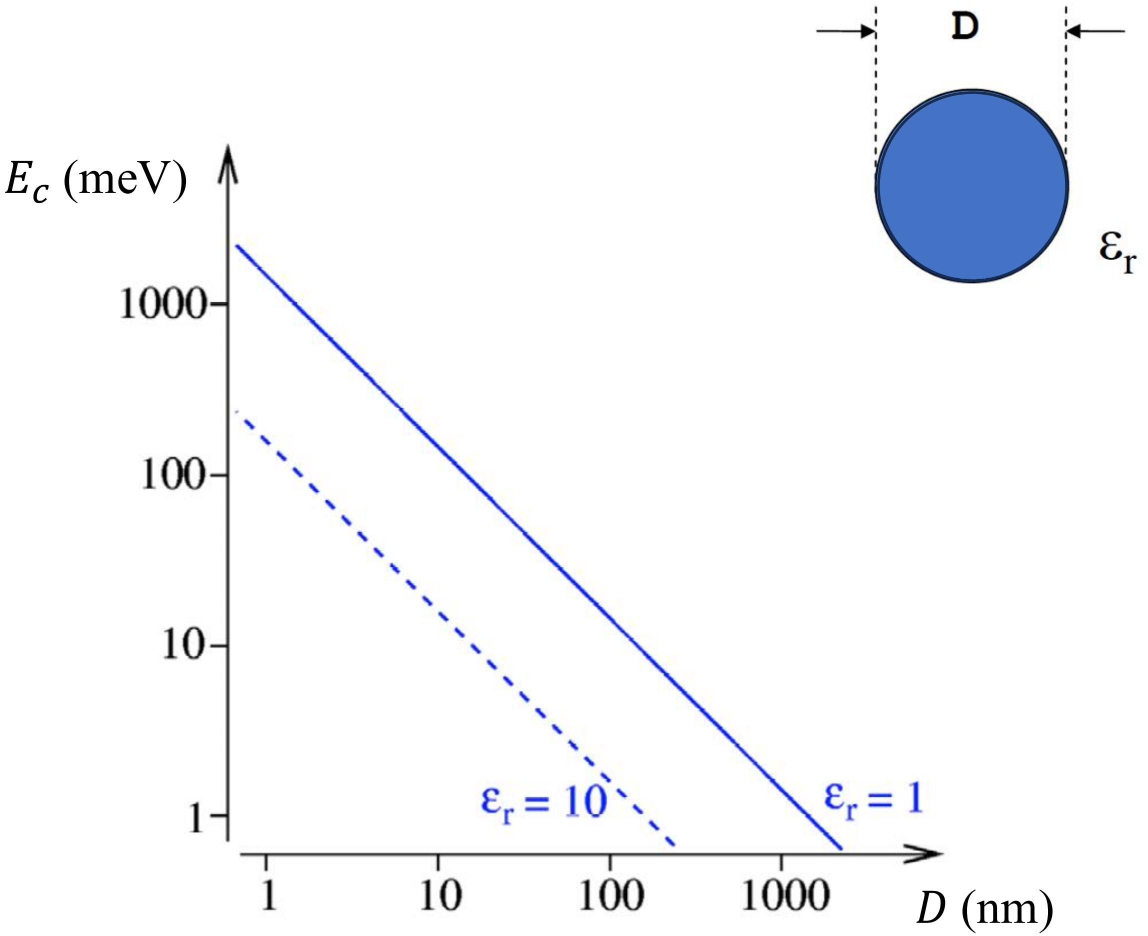
The energy of one additional electron brought onto this dot is given by

$$E_c = \frac{e^2}{2C} = \frac{e^2}{4\pi\varepsilon_0\varepsilon_r D}$$

"charging energy"

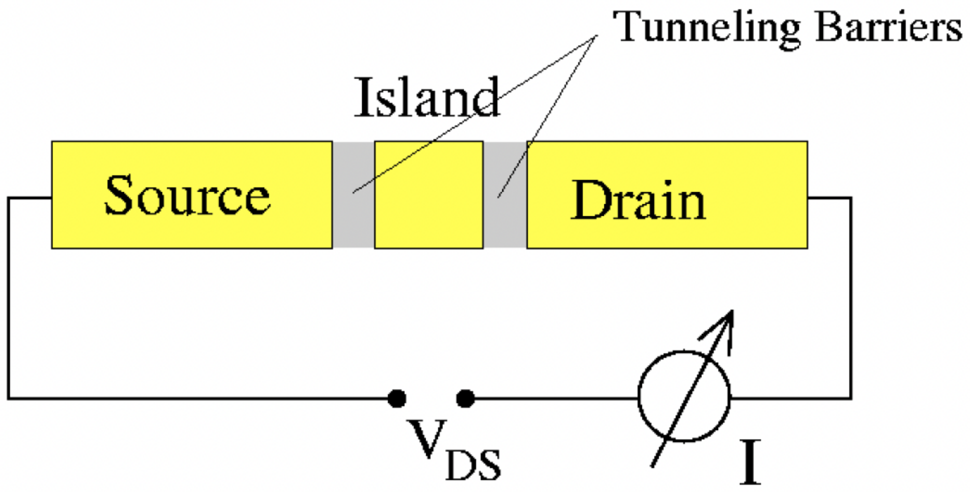
For macroscopic capacitors, this energy is too small to be measured

As the capacitor gets smaller, E_c increases to measurable values



Coulomb blockade in transport

Study addition and removal of electrons onto quantum dots

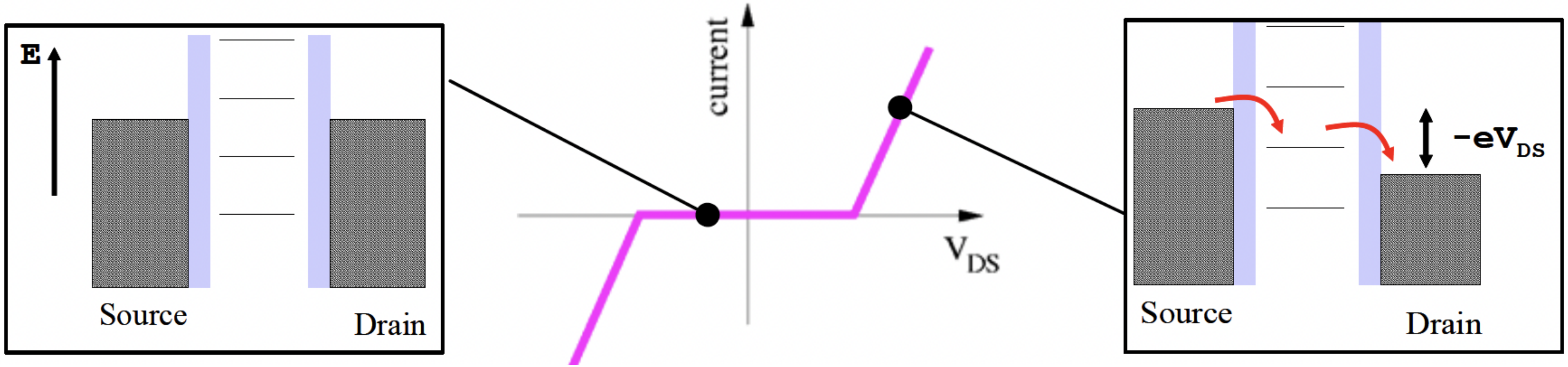


quantum mechanical resonant tunneling

”addition energy spacing” corresponding to E_C

it does not stem from the level quantization due to the quantum confinement

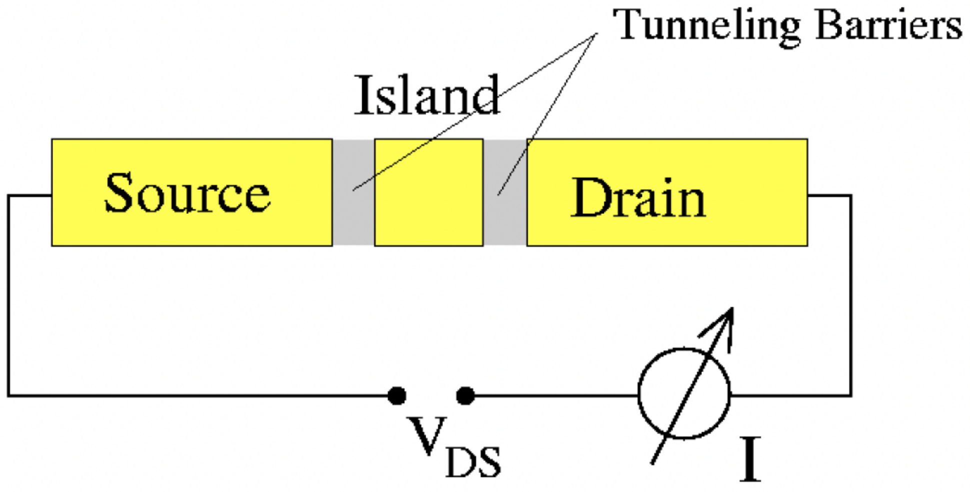
an additional electron can only enter the dot if the Coulomb repulsion is overcome



Blockade

Current flow

When can the Coulomb blockade be seen ?



1. Thermal broadening

$E_C \gg k_B T$

RT: $k_B T \approx 25$ meV

typically, LT investigations (mK to few K)

2. Lifetime

the electron must stay in the dot enough time to be observed

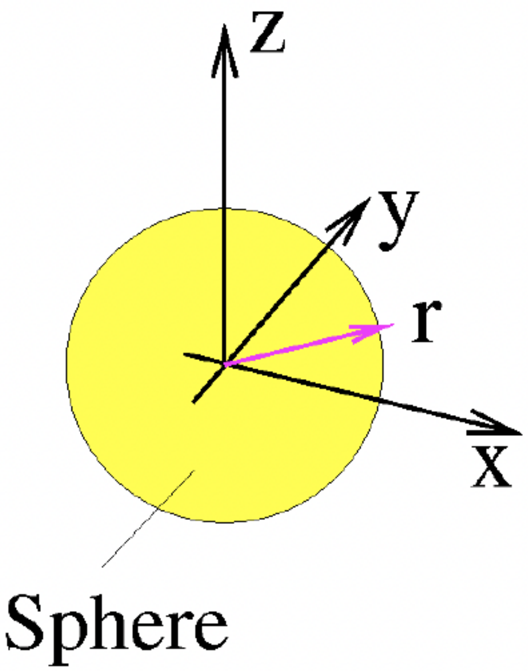
this timescale is related to the resistance R of the tunnel junction, $\Delta t = RC$

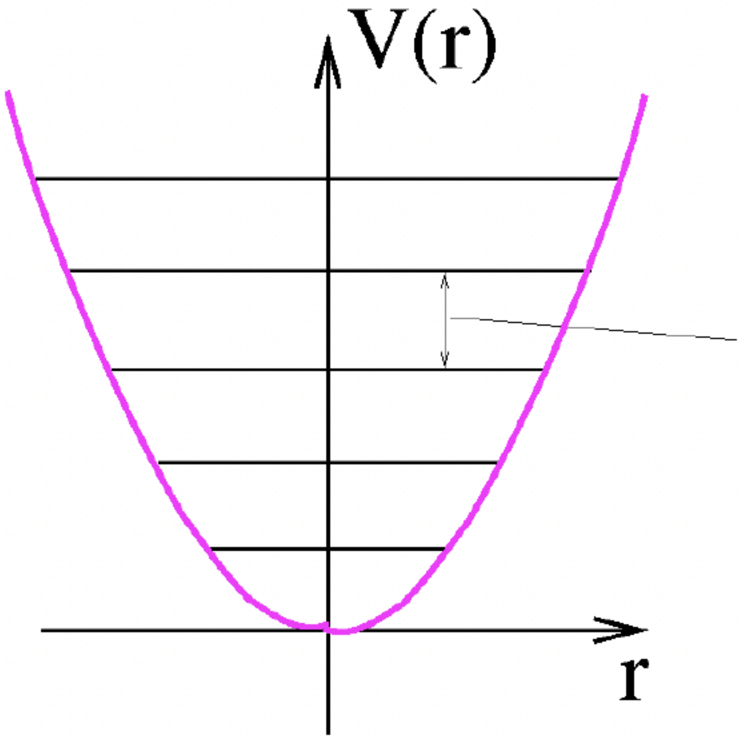
the condition is obtained by the Heisenberg uncertainty principle

$$E_c \, \Delta t \approx \frac{e^2}{C} RC > h \quad \rightarrow \quad R \gg \frac{h}{e^2} \approx 25.8 \text{ k}\Omega$$

parabolic confinement → equally spaced energy levels (harmonic oscillator)

(for rectangular wells: states $\propto n^2$)





$$\Delta = \frac{h^2}{2m^*(2r)^2}$$

When does quantum size effects play a role?

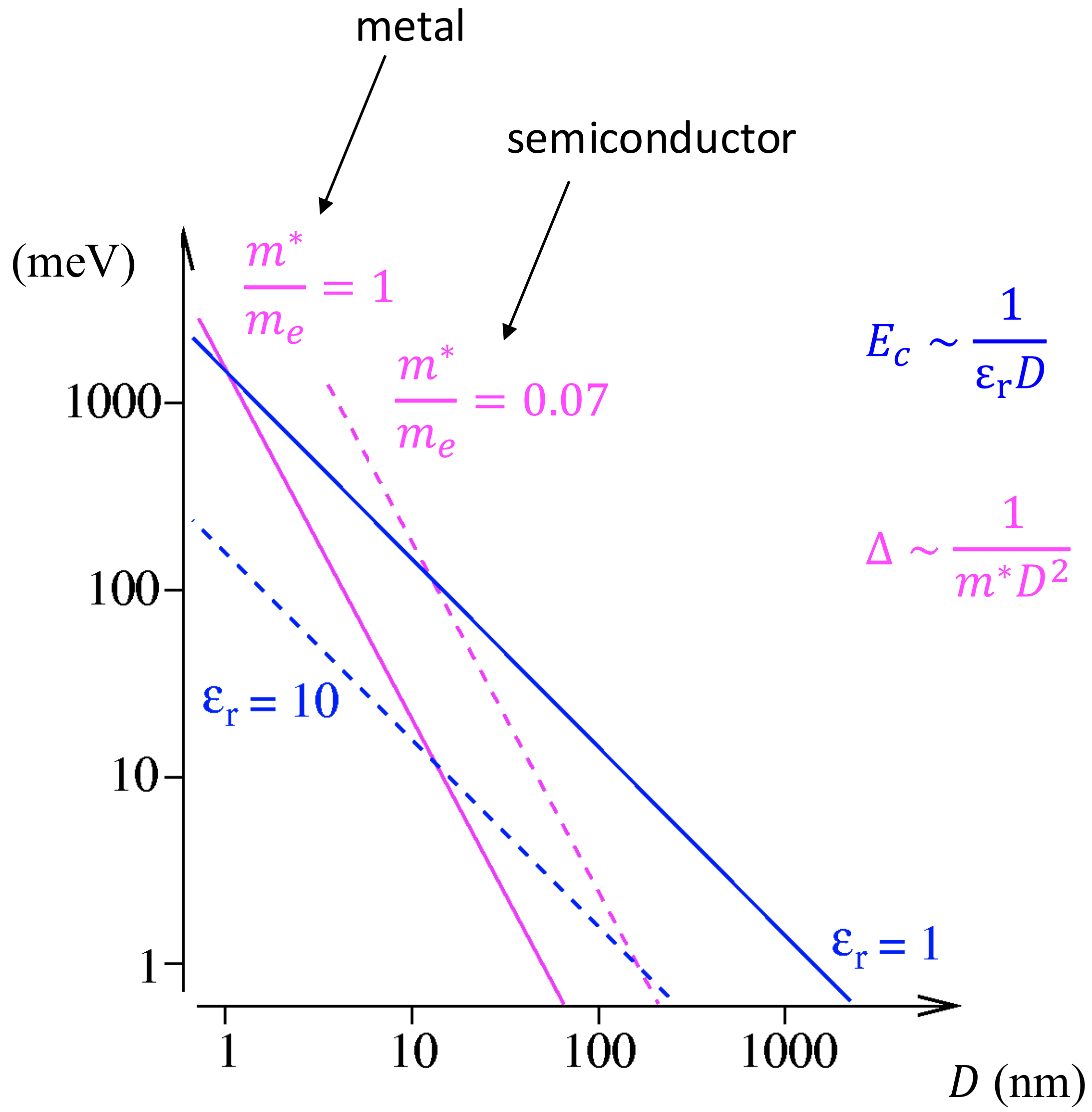
Typical realizations of quantum dots:
small metal islands
semiconductor heterostructures,
electrostatic gates

Quantum size effect (discrete levels): the Fermi wavelength (or the de Broglie wavelength) of the electrons is of the order of the lateral dimension of the quantum dot

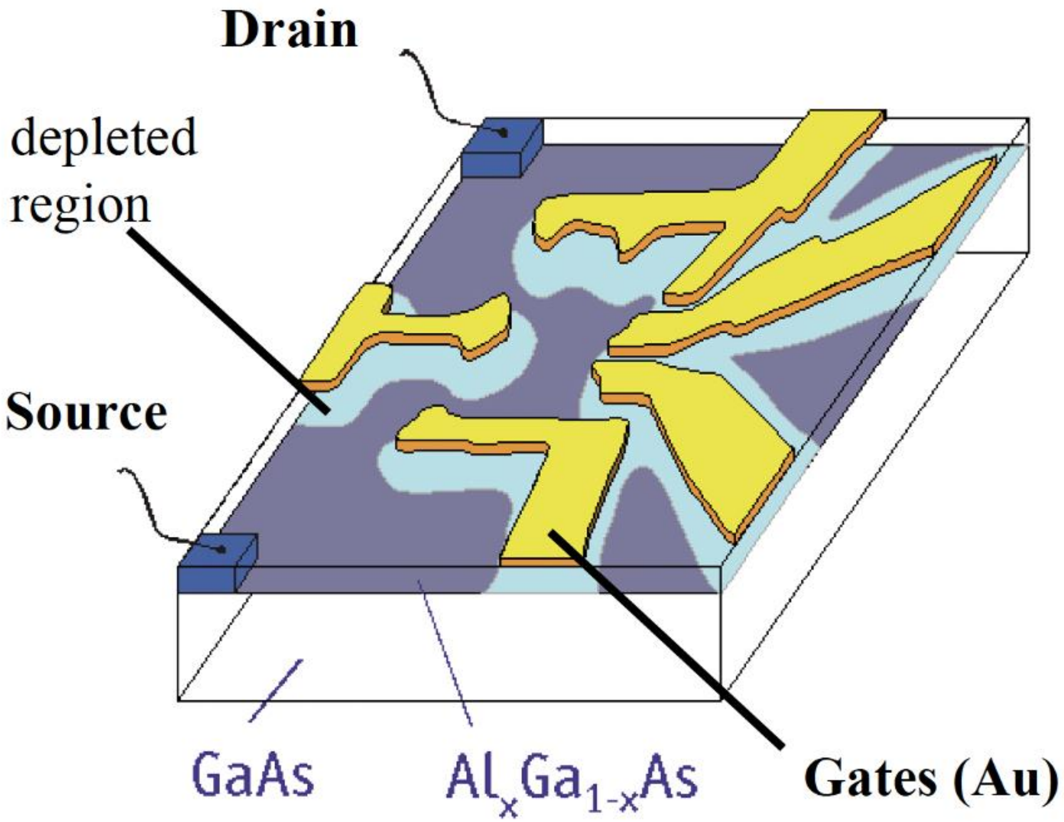
Level splitting for a spherical QD, parabolic confinement : $\Delta = \frac{h^2}{2m^* D^2}$

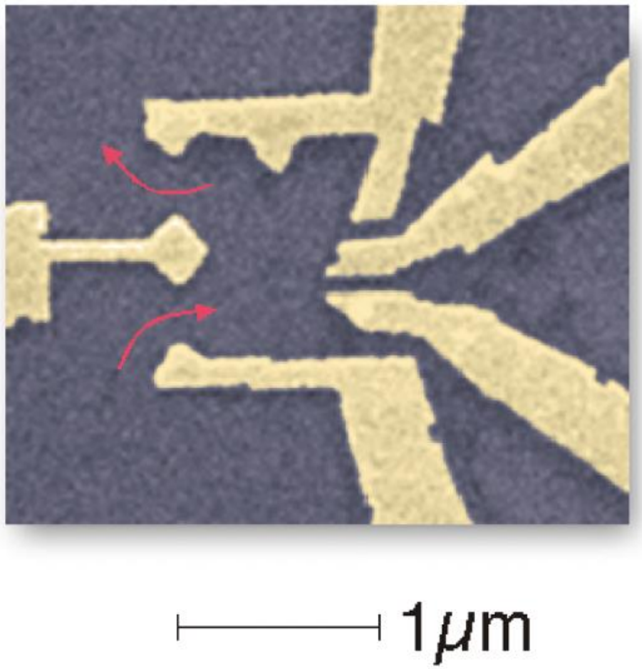
Depending on the nature of the dot, E_c or Δ will be dominant

If $E_c \gg \Delta$: classical dot



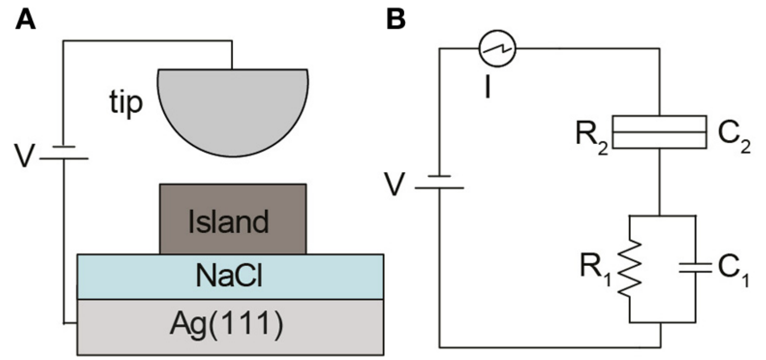
Experimental realization: 2DEG gated heterostructures



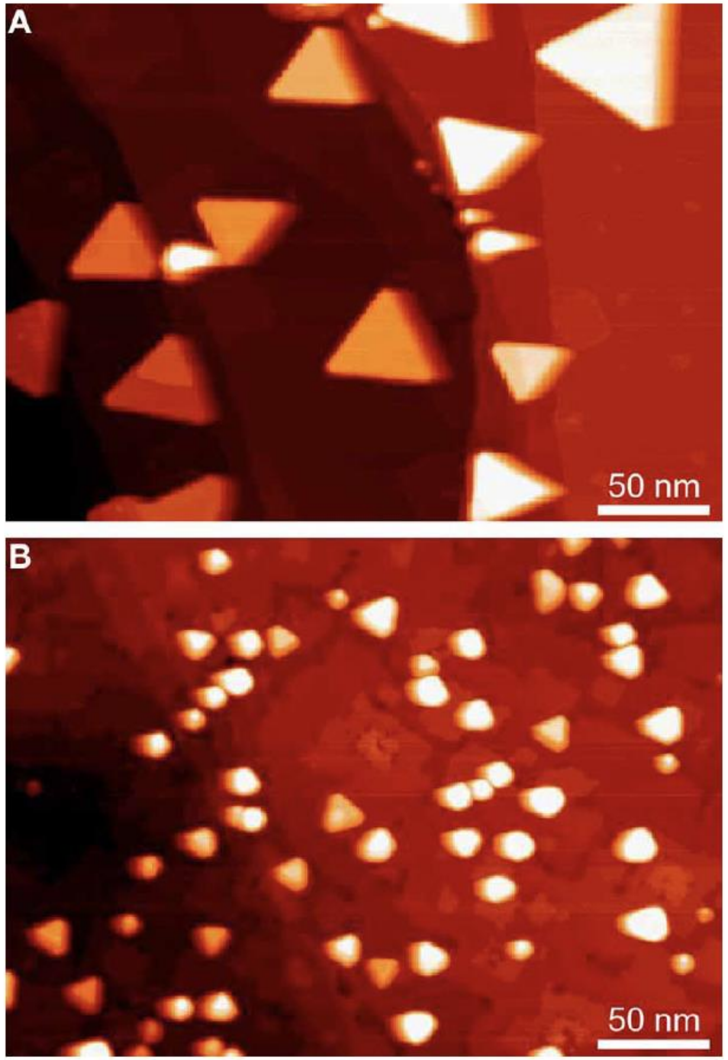


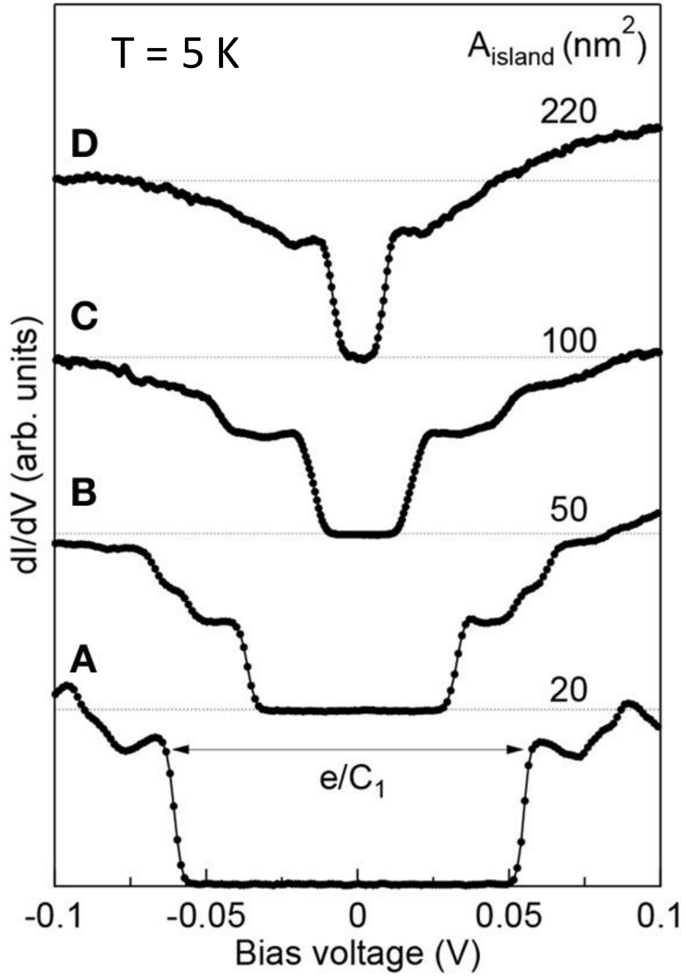
portion of the 2DEG
segregated by the gates

STM experiment



superconducting
gap ≤ 1meV

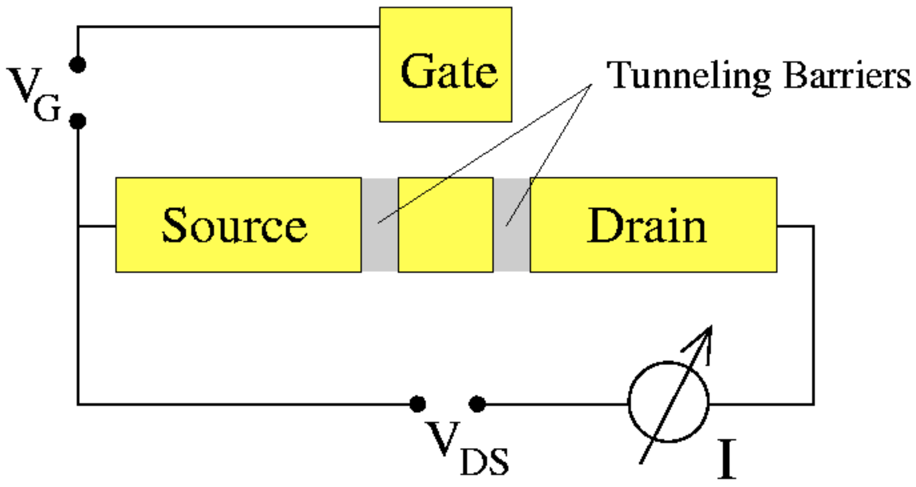


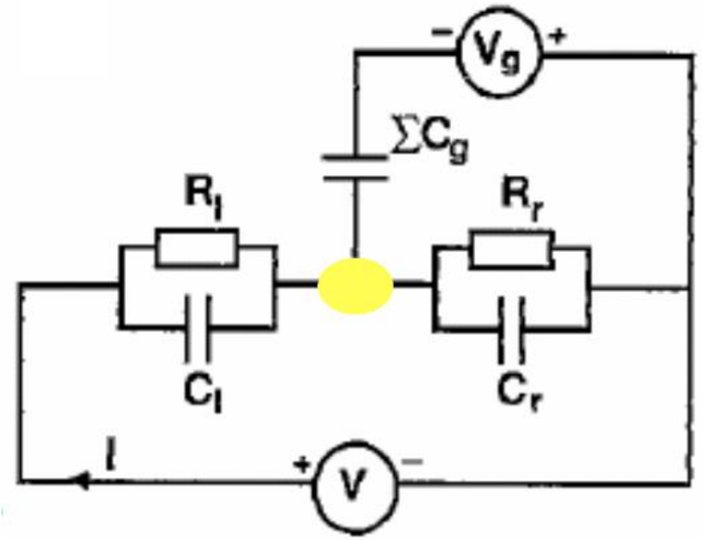


$$C_1 = \varepsilon_0 \varepsilon_r \frac{A_{island}}{d_{NaCl}}$$

Front. Phys. 10.3389/fphy.2013.00013

Single Electron Transistor (SET)



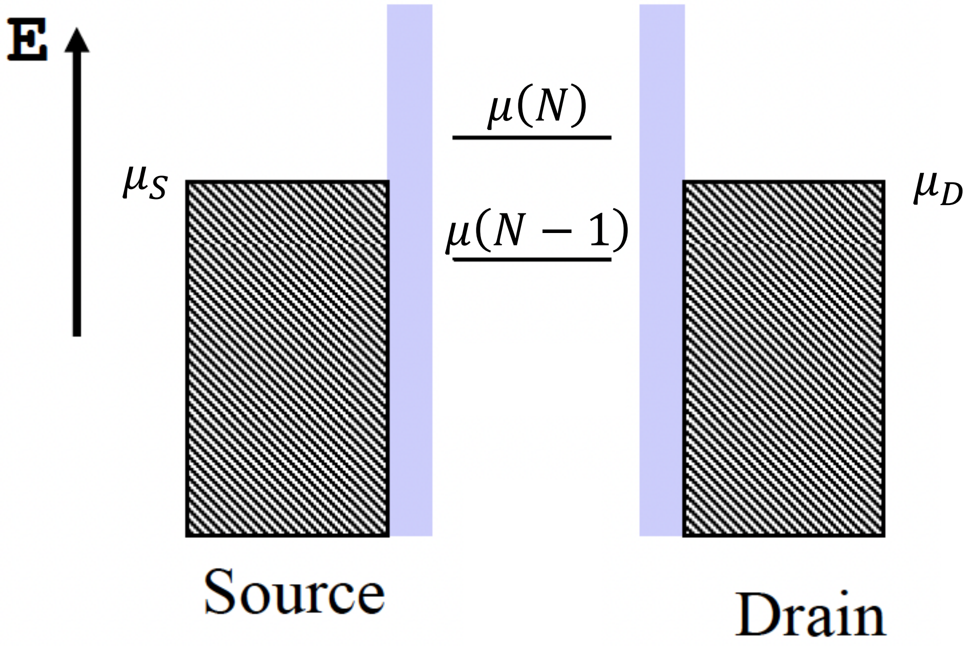


The position of the energy levels within the quantum dot with respect to the electrochemical potential of the leads can be influenced just like in a conventional field effect transistor by an electrostatically coupled gate electrode.

This three-terminal device is called a single electron transistor (SET), since this device can be operated in such a manor that only one electron at a time can pass the quantum dot.

Constant interaction model

1) single capacitance C between electrons on the dot and the environment
2) single-particle energy-level spectrum independent of the number of electrons



Total energy of the dot (CI-model):

$$U(N) = \textit{energy quantized levels + electrostatic energy}$$

Electrochemical potential of N electrons on the dot

$$\mu(N) = U(N) - U(N-1) = E_N + \frac{e^2}{2C}(2(N-N_0)-1) - e\alpha V_G$$

E_N single particle energy level
N_0 number of electrons on the dot without applied voltage
$\alpha = C_G/C$

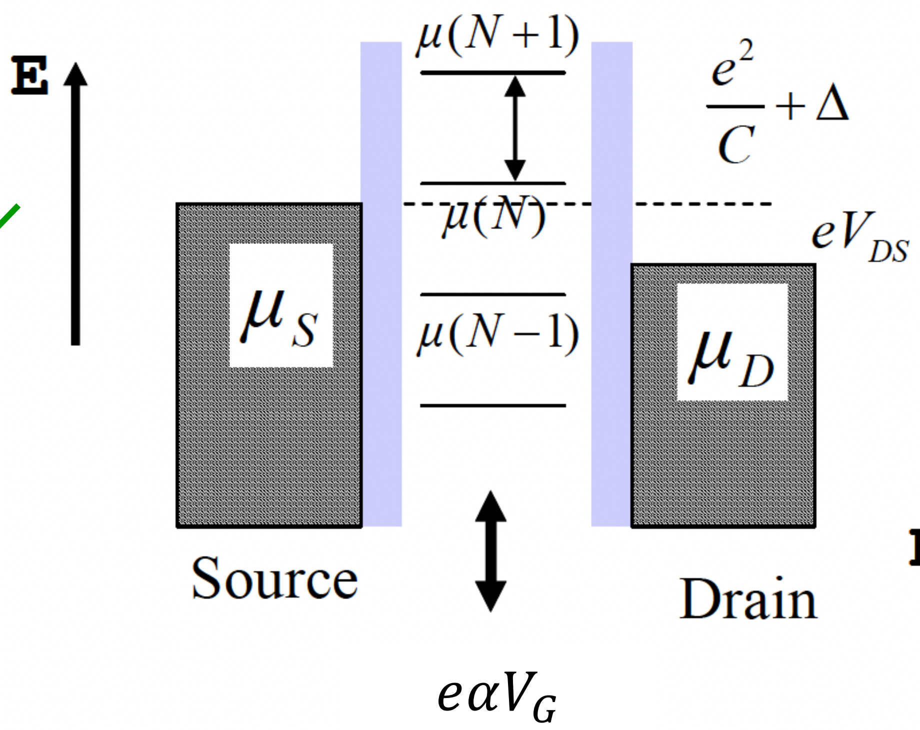
Addition energy (separation between two adjacent energy levels with specific electrochemical potentials)

$$\mu(N+1) - \mu(N) = \frac{e^2}{C} + E_{N+1} - E_N = \frac{e^2}{C} + \Delta$$

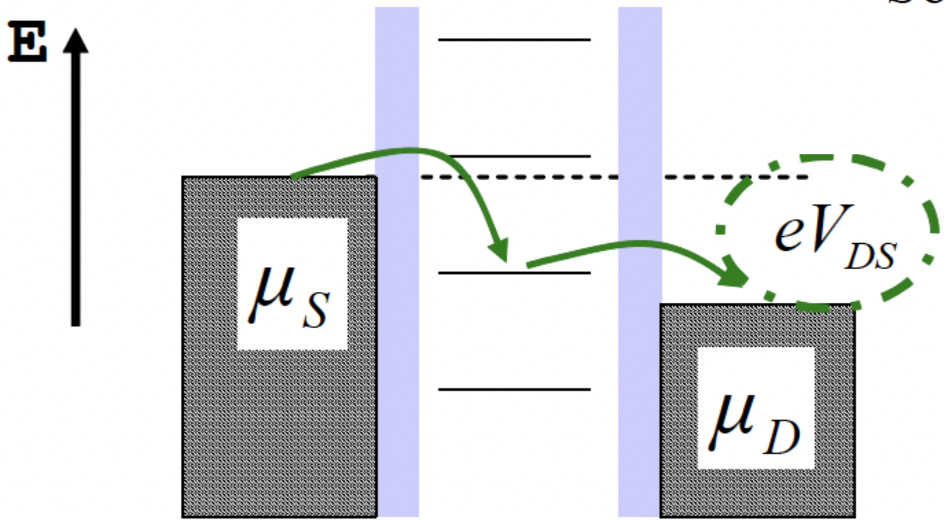
Single electron transport through the dot

electrochemical potential levels of a quantum dot connected via tunnelling barriers to drain and source contacts

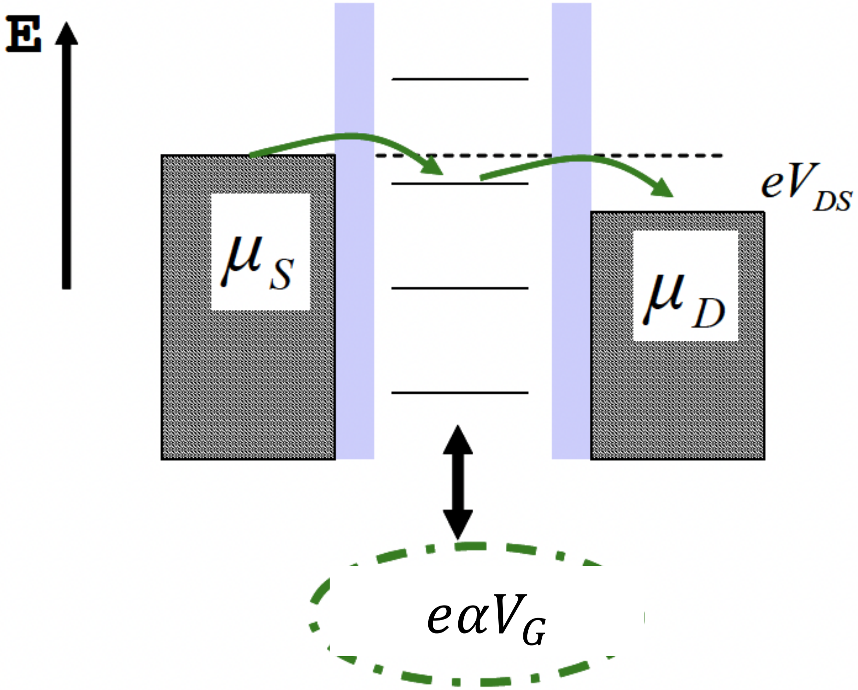
Blockade (off resonance)



Transport by changing V_{DS}



Transport by changing V_G

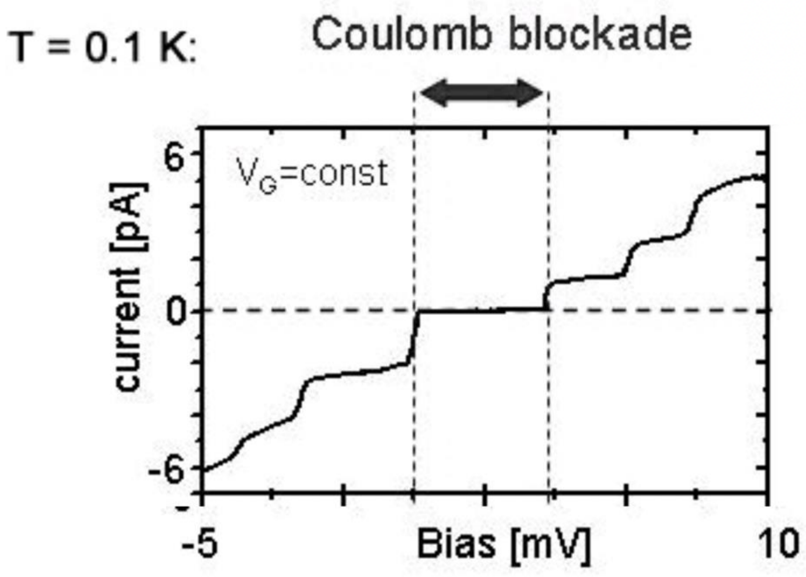


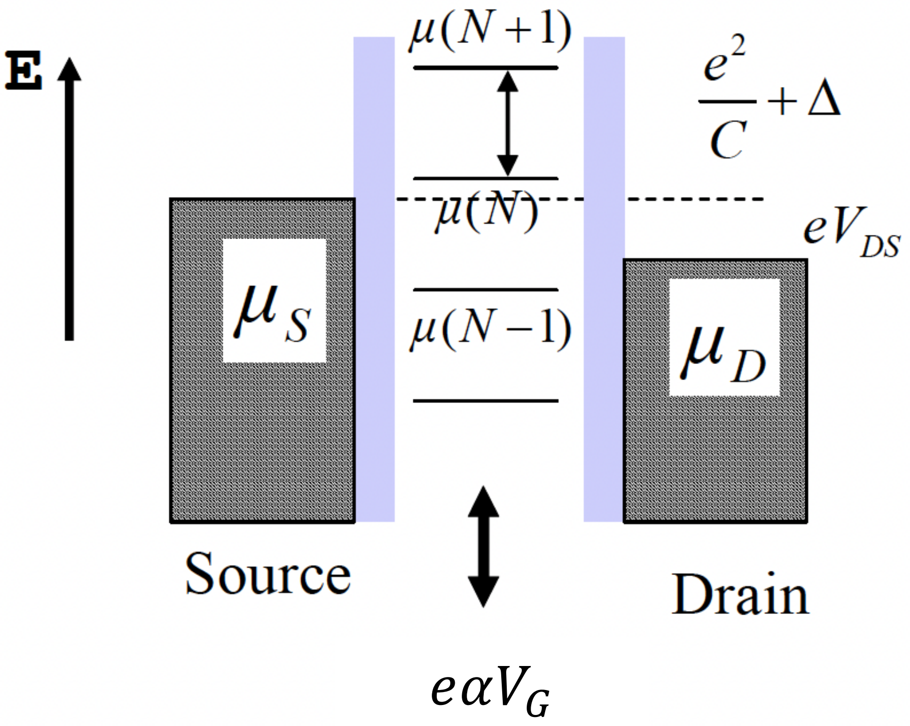
Single electron transport through the dot

Exercises 9.1 – 9.2

Changing the drain-source voltage so that a level lies within the bias window opened by eV_{DS}

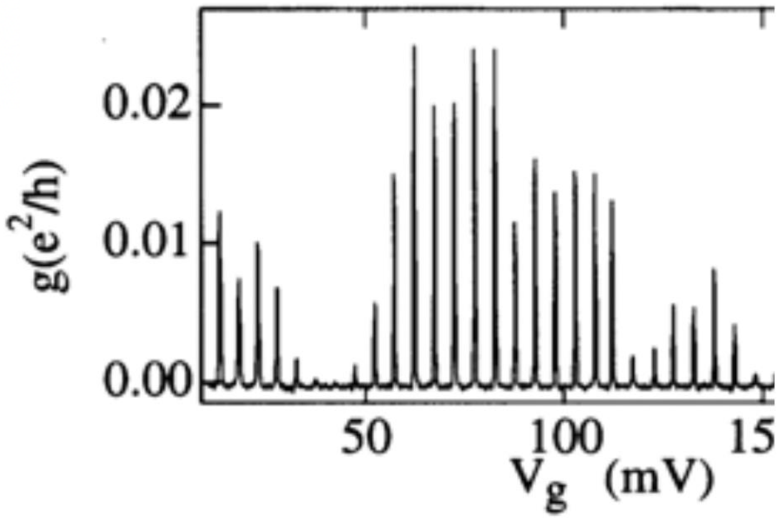
Each time a further level enters the bias window the current suddenly increases





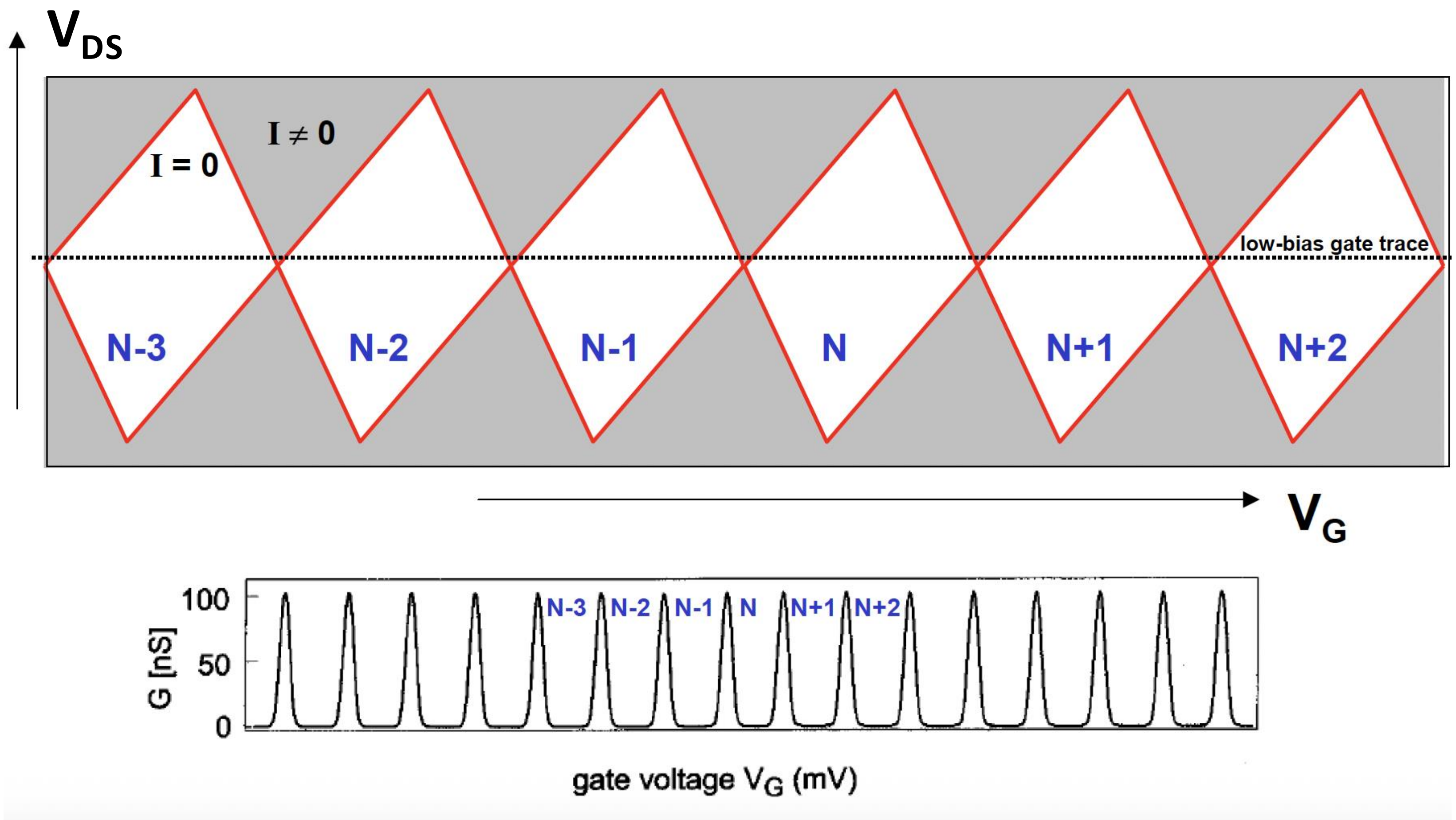
Changing the gate-source voltage so that a level lies within the small bias window eV_{DS}

The conductivity oscillates between a finite value (resonant tunneling) and zero



Coulomb diamond (charge stability diagram)

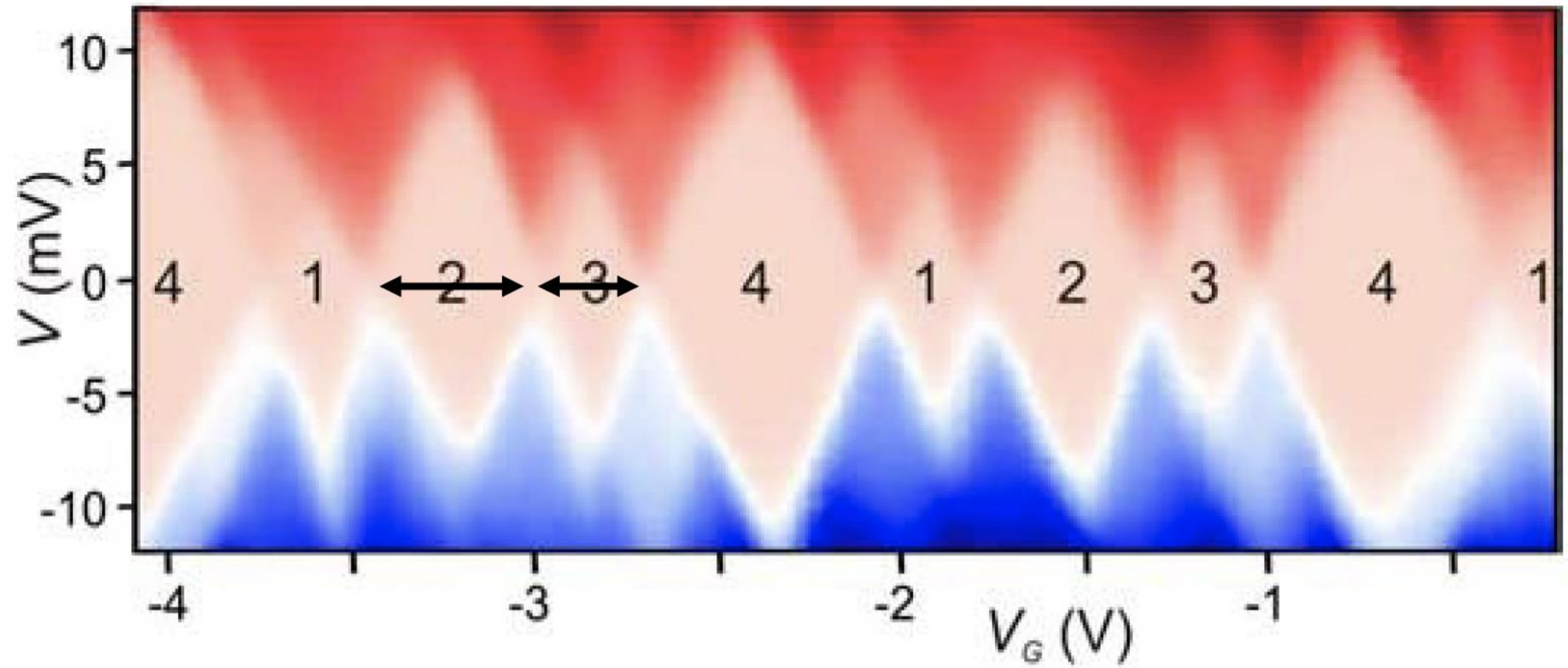
2D graph that plots the conductance (or current) as a function of the gate voltage, V_G, and the bias voltage, V_{DS}, revealing the Coulomb blockade regions and conductance peaks



inside the Coulomb diamonds (white regions): **blockade**, the number of electrons is fixed and no current flows

outside the Coulomb diamonds (grey): **transport**, the number of electrons fluctuates and current flows

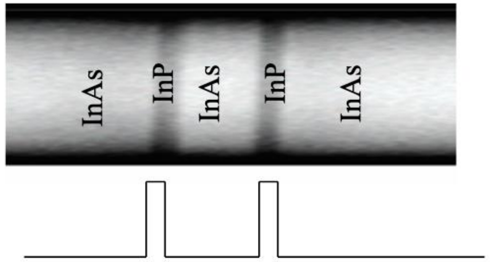
Example: Experimental stability diagram of a carbon nanotube QD



The addition energy is not the same for all additional electrons, there is a "superstructure" (fourfold shell filling, artificial atoms)

phys. stat. sol. (b) **245** (2008) 1455

Example: Single-electron transistors in heterostructure nanowires



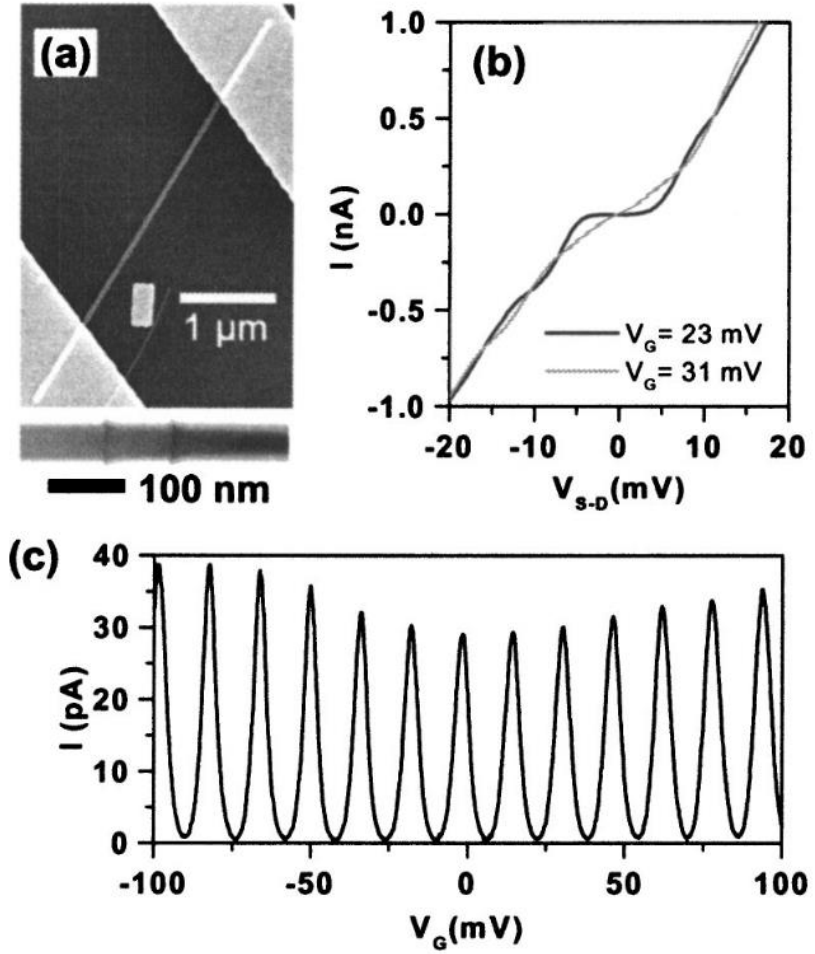
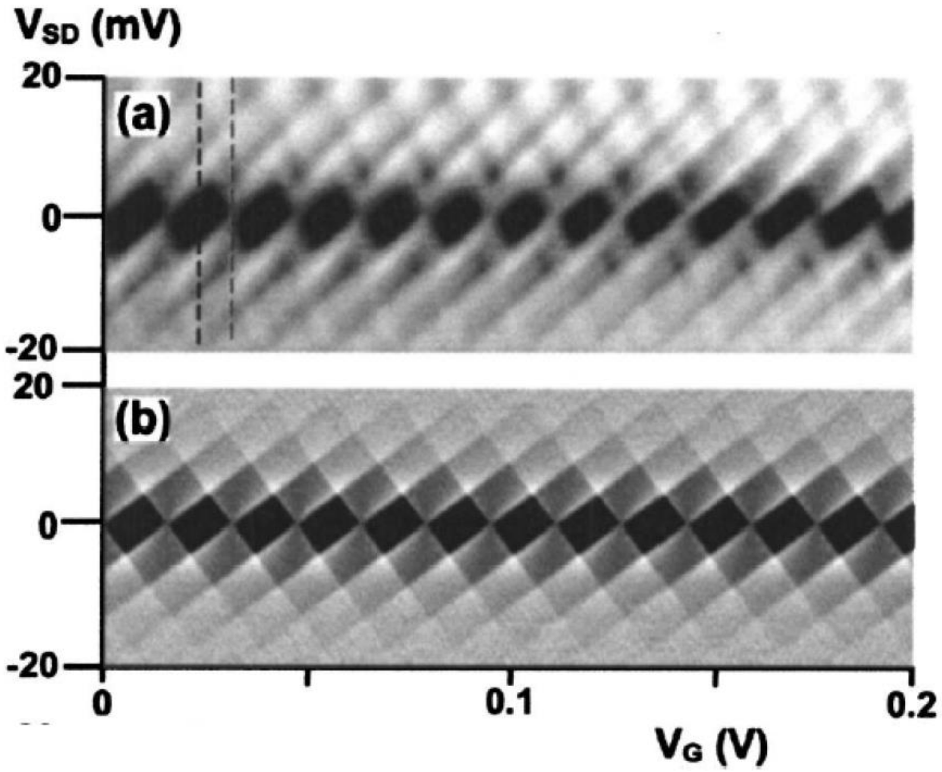


FIG. 2. (a) Scanning electron micrograph of an electrically contacted InAs nanowire with an InP double-barrier structure. The lower inset shows how a selective InAs etch can be used to reveal the InP double barrier (transmission electron micrograph). (b) $I-V$ curves for the 55 nm diameter device in Fig. 2(a) recorded for two different gate voltages at $T=4.2$ K, corresponding to a blocked (black) and a conducting state (gray). (c) Current through the device plotted as a function of the universal back-gate voltage for V_{SD} $=0.5$ mV.

T = 4.2 K 55nm diameter



Appl. Phys. Lett. **83**, 205Z (2003)
https://doi.org/10.1063/1.1606889

SET applications

High sensitivity electrometer (RF-SET for high speed and high sensitivity)

Electrostatic potential probe (the electrochemical potential of the electrons on the dot is very sensitive to electrostatic fields in the close vicinity of the dot)

Single electron memory (like Floating Gate FET)

Thermometer below 1K (based on the width of Coulomb peaks)

...